1. Oblicz granice ciągów:

1. $\lim_{n\to\infty}\frac{\sqrt{n-\sqrt{n}}}{\sqrt[3]{16n^3+2n+1}}$
2. $\lim_{n\to\infty}\frac{2}{\sqrt{4n^2-5}+3n}$
3. $\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^{2n}$
4. $\lim_{n\to\infty}\left(\frac{n+1}{2n}\right)^{n}$

2. Oblicz granice z twierdzenia o trzech ciągach:

1. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n+\sin n}$ $\to e^2$
2. $\lim_{n\to\infty}\frac{\sqrt[n]{2^n+3^n}}{\sqrt[n]{3^n+4^n}}$
3. $\lim_{n\to\infty}\left(1+\frac{1}{n}+\frac{1}{n^2}\right)^{n}$

3. Udowodnij następujące fakty dla ciągów i szeregów o wyrazach dodatnich:

$$\forall n\; x_n \leq y_n \Rightarrow \lim x_n \leq y_n$$

$$\forall n\; x_n \leq y_n \Rightarrow \sum_{n=1}^{\infty} x_n \leq \sum_{n=1}^{\infty} a_n \quad \forall_n\, a_n \leq y_n \Rightarrow \sum_{i=1} a_n \leq \sum_{n=1} a_n$$

$$\forall n\; \sqrt[n]{a_n} \leq r < 1 \Rightarrow \sum_{n=1}^{\infty} a_n \leq \frac{1}{1-r}$$

$$\sum_{n=1}^{\infty}\frac{\left(1+\frac{1}{n}\right)^{n^2}}{3^n} \leq \frac{3}{3-e}$$

4. Obliczyć sumę szeregów:

3 1. $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$ $\sum \frac{1}{n}$ $\quad 2 \quad \sum \frac{1}{n} \quad \sum \frac{1}{n^4}$ $\quad \sum \frac{1}{n^4}$ $\quad 1\cdot 2 = 2$

2. $\sum_{n=1}^{\infty}\ln\left(1+\frac{1}{n}\right)$ (zastosować do ciągu sum częściowych wzór na sumę logarytmów) $\ln\left(\frac{1+n}{n}\right)$

5. Sprawdzić zbieżność szeregów:

1. $\sum_{n=1}^{\infty}\frac{n}{2^n}$
2. $\sum_{n=1}^{\infty}\frac{\ln(n+1)}{2n+1}$ =

$\ln a + \ln b = \ln(ab)$

$\log_c a + \log_c b = \log_c(a\cdot b)$

$\log_b(x) = \frac{\ln(x)}{\ln(b)}$

$\frac{1}{n}$ $\quad \frac{1}{n^2}$

$\lim \sqrt[n]{a} = 1$, dla $a>0$

Maciej Smagór 229789 Analiza 2

1. Oblicz granice ciągów:

1. $\lim_{n\to\infty} \frac{\sqrt{n-\sqrt{n}}}{\sqrt[4]{16n^2+2n+1}} = \lim_{n\to\infty} \frac{\sqrt{n-\sqrt{n}}}{\sqrt[4]{16n^2(1+\frac{1}{8n}+\frac{1}{16n^2})}} =$

$\lim_{n\to\infty} \frac{\sqrt{n-\sqrt{n}}}{2\sqrt{n}\,\sqrt[4]{1+\frac{1}{8n}+\frac{1}{16n^2}}} \cdot \frac{\sqrt{n-\sqrt{n}}}{\sqrt{n-\sqrt{n}}} =$

$\lim_{n\to\infty} \frac{n-\sqrt{n}}{2\sqrt{n(n-\sqrt{n})}} \cdot \frac{1}{\sqrt[4]{1+\frac{1}{8n}+\frac{1}{16n^2}}} =$

$\lim_{n\to\infty} \frac{n(1-\frac{\sqrt{n}}{n})}{2n\sqrt{1-\frac{1}{n}}} \cdot \frac{1}{\sqrt[4]{1+\frac{1}{8n}+\frac{1}{16n^2}}} = \lim_{n\to\infty} \frac{1-\sqrt[3]{n}}{2\sqrt{1-\frac{1}{n}}} \cdot \frac{1}{\sqrt[4]{1+\frac{1}{8n}+\ldots}}$

$= 1$

2. $\lim_{n\to\infty} \frac{2}{\sqrt{4n^2-5}+2n} = \lim_{n\to\infty} \frac{2}{\sqrt{4n^2(1-\frac{5}{4n^2})}+2n} =$

$\lim_{n\to\infty} \frac{2}{2n\sqrt{1-\frac{5}{4n^2}}+2n} = \lim_{n\to\infty} \frac{2}{2n(\sqrt{1-\frac{5}{4n^2}}+1)} =$ $\searrow 0$

3.
$\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^{2n} = \lim_{n\to\infty} \left[\left(\frac{n}{n(\frac{1}{n}+1)}\right)^n\right]^2 = e^2$

4.
$\lim_{n\to\infty} \left(\frac{n+1}{2n}\right)^n = \lim_{n\to\infty} \left(\frac{n(1+\frac{1}{n})}{2n}\right)^n = \lim_{n\to\infty} \left(\frac{1}{2}+\frac{1}{2n}\right)$

$\lim_{n\to\infty} \left[\frac{1}{2}\left(1+\frac{1}{n}\right)\right]^n = 0$

2. Oblicz granicę z twierdzenia o trzech ciągach.

1.
$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{2n+\sin n}$

(1+1/n)^2n+1 $> \left(1+\frac{1}{n}\right)^{2n+\sin n}$ > (1+1/n)^2n−1

3. $\lim_{n\to\infty} \left(1+\frac{1}{n}+\frac{1}{n^2}\right)^n$

$$e \leq \left(1+\frac{1}{n}+\frac{1}{n^2}\right)^n$$

4. Obliczyć sumę szeregów: Nieprawidłowo

1.
$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \frac{1}{n} \sum_{n=1}^{\infty} \frac{1}{n+1} = \sum_{n=1}^{\infty} \frac{1}{n} \sum_{n=2}^{\infty} \frac{1}{n} = 2 \cdot 1 = 2$$

2.
$$\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n}\right)$$

$$\ln 2 + \ln\frac{3}{2} + \ln\frac{4}{3} + \ln\frac{5}{4} \ldots =$$
$$\ln \cancel{2} \cdot \frac{\cancel{3}}{\cancel{2}} \cdot \frac{\cancel{4}}{\cancel{3}} \ldots = \ln 1 = 0$$

$$S_1 = \ln\left(1+\frac{1}{n}\right) = \ln 2$$
$$S_2 = \ln 2 + \ln\frac{3}{2} = \ln 3$$
$$\vdots$$
$$S_n = \ln(n+1)$$

5. Sprawdzić zbieżność szeregów:

1. $\sum_{n=1}^{\infty} \frac{n}{7^n}$ Z kryterium D'Alemberta:

$$\lim_{n\to\infty} \frac{n+1}{7^{n+1}} \cdot \frac{7^n}{n} = \lim_{n\to\infty} \frac{n\left(1+\frac{1}{n}\right)}{7^n \cdot 7} \cdot \frac{7^n}{n} = \lim_{n\to\infty} \left(1+\frac{1}{n}\right) \cdot \frac{1}{7} = ($$

$\sum_{n=1}^{\infty} \frac{n}{7^n}$ jest zbieżny

2.